ARTICLE NO. 80-4032

Multiple Body Six Degrees of Freedom Potential Flow Pressure Equation

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The development of an equation which efficiently and credibly expresses pressure for arbitrary bodies, each moving with six degrees of freedom, is presented. Using an arbitrary number of unit velocity potential distributions, a form of the nonsteady Bernoulli equation is derived and transformed using indexing methods to yield an expression for pressure which virtually eliminates algebraic development and computer coding errors associated with calculations of pressure on bodies where flow is modeled with many unit velocity potential functions. Pressure and hydrodynamic coefficients are expressed directly in terms of the potentials and their variations. Numerical verification of the equation is provided by comparison of hydrodynamic coefficients calculated for a test body using two different potential flow computer codes which contain the new pressure algorithm and an alternative method for simple bodies. The method reduces the complexity of developing algorithms and software to model missile launch from submarines using potential flow theory for the combined motions of the two bodies. The algorithm is general and may be used with an arbitrary number of unit velocity potentials.

Nomenclature

A_p = patch area
C_{ij} = steady-state hydrodynamic pressure coefficients
CD_i = nonsteady hydrodynamic pressure coefficients
e_i = orthogonal unit vectors collinear with moving reference frame axis
e_{i_s} = orthogonal unit vectors collinear with moving reference frame axis on submarine
e_{i_m} = orthogonal unit vectors collinear with moving reference frame axis on missile
f_{n1n2_i} = mathematical function, defined in text
f_{ω_i} = mathematical function, defined in text
E_i = unit vectors collinear with inertial reference frame axes
n = number of unit velocity functions
N = number of unique pressure coefficients
N_k = unit vectors on submarine or missile surface patch
p = pressure at an arbitrary point in fluid
p_p = pressure at a fixed point on a body
R_c = position vector to reference point fixed on moving coordinate system
u_i = moving frame velocities (translational and rotational)
V_f = inertial velocity of fluid
V_p = inertial velocity of point fixed on moving body
V_{p_m} = inertial velocity of point fixed on missile
V_{p_s} = inertial velocity of point fixed on submarine
X_i = Cartesian coordinates
X_{im} = Cartesian coordinate system on missile
X_{i_s} = Cartesian coordinate system on submarine
$X_{i,j}$ = patch direction cosines between axes of arbitrary orthogonal coordinate system on patch and moving frame coordinate system
Z_i = submarine coordinate system quantities describing relative positions between two bodies
ρ = position vector to body surface from body reference frame
ρ_f = fluid density (const)
ϕ_i = unit velocity potential
$\phi_{j,k}$ = spatial variation of unit velocity potential
Φ = velocity potential function
Φ_p = velocity potential function at fixed point on moving frame
ω = angular velocity vector of moving frame

Received Aug. 10, 1979.

Index categories: Hydrodynamics; Computational Methods; Marine Hydrodynamics.

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I. Introduction

DYNAMIC analysis of underwater launched fleet ballistic missiles requires modeling of the hydrodynamic forces acting on the missile while emerging from a submarine. Many current industry methods use slender body and/or potential flow theory to model hydrodynamic forces. The modeled bodies are typically symmetric about the longitudinal axis. Much simplification results from assuming that the missile is moving in an infinite fluid. Current methods to model the underwater dynamics of underwater submarine-launched missiles predominately use infinite fluid symmetric body assumptions in potential flow modeling, even in close proximity to the submarine.

There is substantial interest in improved hydrodynamic modeling of bodies emerging from other bodies. A current operational method for modeling arbitrary emerging bodies uses a technique of image planes.[1] Very little difference is predicted between hydrodynamic forces using truncated infinite fluid coefficients and that using rather rigorously determined emerging body coefficients for FBM missile configurations where the motions are essentially perpendicular to the infinite plane (Figs. 1 and 2). The dominant hydrodynamic force on the missile nose is insensitive to the presence of the boundary conditions far removed from the major lift.

There is interest in the use of nonsymmetric bodies for use as test vehicles for underwater launch testing. The hydrodynamic modeling of nonsymmetric bodies somewhat increases the complexity and greatly increases the cost of potential flow solutions. When the body is considered to be emerging from or near another body (the submarine), the modeling problem is then extremely complex due to the large amount of numerical computations required. Advantages of

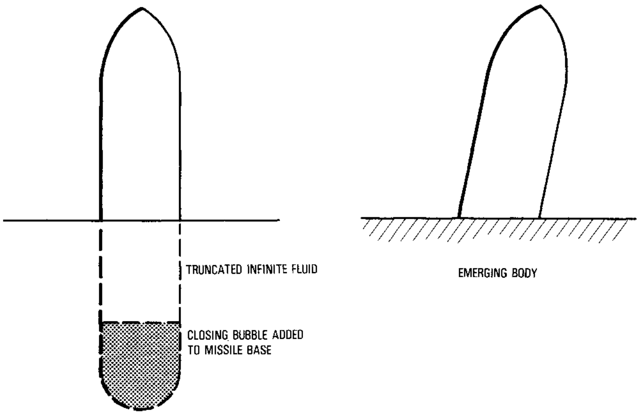

Fig. 1 Emerging body hydrodynamic modeling.

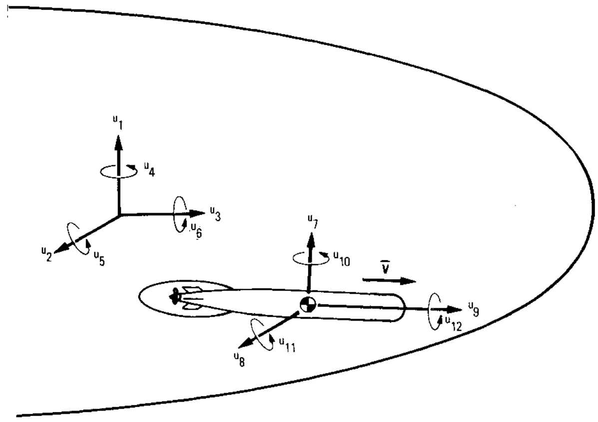

Fig. 3 General two body problem.

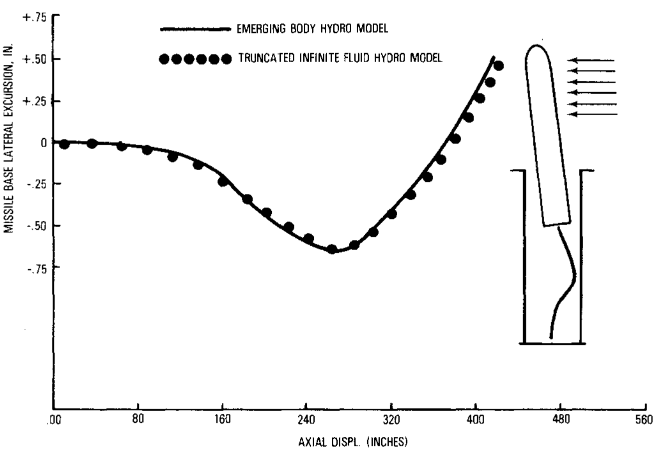

Fig. 2 Comparison of two hydrodynamic models.

symmetry are lost and potential flow solutions must be created for many different relative positions between the bodies.

The advent of greatly improved potential flow solution methods makes it feasible to use potential flow theory to model the hydrodynamic forces on complex bodies undergoing complex motions.[2] The method of Chow has been extensively investigated for applicability to potential flow problems where flexibility is desirable between computing cost and theoretical precision. Chow's method has been demonstrated to be quite insensitive to missile surface characterizations and is feasible for use in potential flow problems which require many solutions.[3]

The potential flow problem solution methodology is essentially invariant to the number of degrees of freedom the body or bodies may have. If energy principles could be used to evaluate hydrodynamic forces and moments, the procedure required to determine hydrodynamic coefficients from the potential flow solutions would be straightforward.

For the general case of a body emerging from another body, however, there appears to be no alternative but to use the Bernoulli equation to describe total force and moment on the emerging body via integration of pressure. It is the expansion of the Bernoulli equation, using moving coordinates and many degrees of freedom, which can create a significant algebra problem, not only because of conceptual difficulty, but by virtue of the sheer volume of arithmetic manipulations which must be performed and coded.

An arbitrary body moving with six degrees of freedom has 162 coefficients describing total force and moment. Modeling of distributed loading or pressure requires distributed characterization of each of the coefficients. Application of Euler's equations to determine pressure on an arbitrarily moving body is complicated when six or more unit velocity potentials exist in the potential function. If a direct method is used to accomplish these operations, the task is tedious for the six degree of freedom (DOF) case. It is impractical for the 12 DOF case. Experience with 3 and 6 DOF hydrodynamic modeling indicates that it is difficult to assure elimination of development or software coding errors when expanding the Bernoulli equation with direct algebraic manipulation to separate all unique combinations of the generalized velocities.

Verification of hydrodynamic coefficient algorithms and software is difficult due to the lack of analytical solutions, except for simple bodies which have but few nontrivial coefficients. Errors in development or software can easily remain undetected when analytical solutions are trivial.

This development effort was inititated to fill the need for an efficient method to determine hydrodynamic coefficients of arbitrarily shaped bodies. The method was originaly derived for arbitrary bodies moving with six degrees of freedom in an infinite fluid. This allowed validation of hydrodynamic coefficients by using three DOF coefficient algorithms determined through alternaive methods. The method has been extended to the more general problem of an arbitrary body emerging from another body. The algorithm is general and may be used for an arbitrary number of unit velocity potential functions.

II. Formulation

Problem Description

The objective is to develop an algorithm that cleanly expresses the hydrodynamic pressure on a moving body surface using a family of solutions for each of the n-unit velocity potentials ($n = 12$ for this problem).

$$p_p = p_p(u_i, \dot{u}_i, Z_j) \qquad i = 1,n; j = 1,6 \tag{1}$$

That is, for a given relative position between the submarine and missile, we wish to express pressure at every point as a function of the velocities and accelerations of both bodies u_i and $\dot{u}_i$, $i = 1$, n. The motions of points on the bodies may be described with the scalers u_i, Fig. 3.

$$V_{P_s} = \sum_{i=1}^{3} u_i e_{i_s} + \sum_{i=4}^{6} u_i e_{i-3_s} \times X_{i-3_s} \tag{2}$$

$$V_{P_m} = \sum_{i=7}^{9} u_i e_{i-6_m} + \sum_{i=10}^{12} u_i e_{i-9_m} \times X_{i-9_m} \tag{3}$$

Given solutions for the potential function Φ, we now strive to apply the Bernoulli equation to compute pressure.

A derivation of a particular form of the Bernoulli equation will be described to assure confidence in the governing equation used to develop the pressure algorithm. There are two reasons for presenting a derivation of the Bernoulli equation: 1) there can be question as to the correct form of the equation to use for a problem of moving coordinates in an initially still fluid, and 2) it is desirable to identify a form of the equation which is convenient for the development of the pressure algorithm to follow.

The Bernoulli equation will be examined and transformed using vector notation to yield an algorithm in terms of the numerical quantities conveniently available after generating families of solutions for the n-unit velocity potentials.

We begin with the following quantities as knowns for *each* patch defining the body surfaces: $A_p, X_i, X_{i,j}$, and ϕ_i.

Note that the unit velocity potential ϕ_i has dependence on the relative position between the missile and submarine.

$$\phi_i = \phi_i(Z_j) \qquad j = 1,6$$

III. Bernoulli Equation for Translating and Rotating Coordinate System

Ignoring gravitational and hydrostatic effects which may be modeled using conventional means, the inertial acceleration of a fluid particle may be given by:

$$-\nabla p = \rho_f \frac{DV_f}{Dt} \tag{4}$$

The fluid inertial velocity is given by the velocity potential with

$$V_f = \nabla \Phi \tag{5}$$

where

$$\Phi = \sum_{i=1}^{n} u_i(t)\phi_i(X_j) \qquad j=1,3 \tag{6}$$

Combining Eqs. (4) and (5) gives

$$-\nabla p = \rho_f \nabla \frac{D\Phi}{Dt} \tag{7}$$

Use of the chain rule in Eq. (7) gives

$$-\nabla p = \rho_f \nabla \left[\sum_{i=1}^{3} \frac{\partial \Phi}{\partial X_i}\frac{\partial X_i}{\partial t} + \frac{\partial \Phi}{\partial t} \right] \tag{8}$$

where $\partial\Phi/\partial t$ is time variation as perceived from the inertial reference frame.

Taking the gradient of both sides of Eq. (8) gives:

$$-\sum_{k=1}^{3} \frac{\partial p}{\partial X_k} e_k = \rho_f \sum_{k=1}^{3} \left[\sum_{i=1}^{3} \frac{\partial^2 \Phi}{\partial X_i \partial X_k} + \frac{\partial^2 \Phi}{\partial X_k \partial t} \right] e_k \tag{9}$$

Since

$$\frac{\partial X_i}{\partial t} = \frac{\partial \Phi}{\partial X_i} \tag{10}$$

and

$$\frac{\partial}{\partial X_k} \frac{(\partial\Phi/\partial X_i)^2}{2} = \frac{\partial \Phi}{\partial X_i} \frac{\partial^2 \Phi}{\partial X_i \partial X_k} \tag{11}$$

Eq. (9) may be written

$$-\sum_{k=1}^{3} \frac{\partial p}{\partial x_k} e_k = \rho_f \sum_{k=1}^{3} \left[\sum_{i=1}^{3} \frac{1}{2} \frac{\partial}{\partial X_k} \left(\frac{\partial \Phi}{\partial X_i}\right)^2 + \frac{\partial}{\partial X_k}\left(\frac{\partial \Phi}{\partial t}\right) \right] e_k \tag{12}$$

Equation (12) may be derived by taking the gradient of both sides of

$$-p = \rho_f \left[\sum_{i=1}^{3} \frac{1}{2} \left(\frac{\partial \Phi}{\partial X_i}\right)^2 + \frac{\partial \Phi}{\partial t} \right] + C(t) \tag{13}$$

where $C(t)$ is an arbitrary function of time.

Dropping the time dependence and using the dot product, Eq. (13) becomes

$$-p = \rho_f [\tfrac{1}{2} \nabla\Phi \cdot \nabla\Phi + (\partial\Phi/\partial t)] \tag{14}$$

The $\partial\Phi/\partial t$ term represents variation in the potential function as perceived from the inertial frame. From the definition of the potential function using moving coordinates, it is apparent that it is desirable to be able to express the time variation as perceived from the moving frame. We accomplish this by transforming the $\partial\Phi/\partial t$ term using the total potential function time derivative referred to the inertial frame

$$\frac{D\Phi_p}{Dt} = \sum_{i=1}^{3} \frac{\partial \Phi}{\partial X_i}\dot{X}_i + \frac{\partial \Phi}{\partial t} \tag{15}$$

to get from Eq. (14)

$$-p = \rho_f \left[\frac{1}{2} \nabla\Phi \cdot \nabla\Phi - \sum_{i=1}^{3} \frac{\partial \Phi}{\partial X_i}\dot{X}_i + \frac{D\Phi_p}{Dt} \right] \tag{16}$$

where

$$\dot{X}_i = (\dot{\bar{R}}_c + \omega \times \rho) \cdot E_i \tag{17}$$

Use of Eq. (17) in Eq. (16) yields

$$-\frac{p}{\rho} = \nabla\Phi \cdot \left(\frac{\nabla\Phi}{2} - \dot{\bar{R}}_c\right) + \frac{D\Phi_p}{Dt} - \nabla\Phi \cdot (\omega \times \rho) \tag{18}$$

Equation (18) represents the Bernoulli equation in a form that we can use to develop a pressure algorithm. It may be transformed for comparison with other forms to note that it is different from that given by Lamb[4] for a moving body with moving coordinates. The equation in Ref. 4 does not include stagnation pressure.

IV. Pressure Algorithm

Pressure, force, and moment are conveniently expressed with coefficients multiplied by appropriate kinetic quantities. Pressure may be expressed as

$$p = \sum_{i=1}^{n} \left[\sum_{j=1}^{n} u_i u_j C_{ij}(Z_K) + \dot{u}_i(Z_k) \right] \qquad k = 1,6 \tag{19}$$

After using the symmetrical properties of the C_{ij} array, there are a number of unique coefficients for a particular $\{Z\}$, defined with

$$N(n) = \frac{n(n+1)}{2} + n \tag{20}$$

For a single body with six degrees of freedom ($n=6$)

$$N_6 = 27 \tag{21}$$

and for our twelve degrees of freedom case

$$N_{12} = 90 \tag{22}$$

Calculation of hydrodynamic coefficients is accomplished by integration of pressure over the body to yield coefficients for the six forces and moments. For the general two-body case, there are 540 unique coefficients.

The C_{ij} array is determined from Eq. (18) using Eq. (6). Consider the $\vec{R}_c - (\nabla\Phi/2)$ term in Eq. (18). The inertial velocity of the origin of the moving coordinate system may be represented as the gradient of a scaler function defined as

$$R_c = \nabla\Phi_c \tag{23}$$

where

$$\Phi_c = \sum_{i=1}^{3} u_i X_i \tag{24}$$

Then, using Eqs. (6), (23), and (24)

$$\left(\dot{R}_c - \frac{\nabla\Phi}{2}\right) = \nabla\left[\sum_{i=1}^{3} u_i X_i - \frac{1}{2}\sum_{i=1}^{n} u_i \phi_i\right] \tag{25}$$

Evaluation of Eq. (25) requires variation of ϕ_i in three arbitrary, but orthogonal directions. A coordinate system is located on each patch with one axis on the patch normal.

Taking the gradients in Eq. (25) gives

$$\left(\dot{R}_c - \frac{\nabla\Phi}{2}\right) = \sum_{i=1}^{3} u_i \sum_{k=1}^{3} X_{i,k} N_k - \frac{1}{2}\sum_{i=1}^{n} u_i \sum_{k=1}^{3} \Phi_{i,k} N_k \tag{26}$$

where

$$X_{i,k} = \frac{\partial X_i}{\partial N_k} \tag{27}$$

and

$$\phi_{i,k} = \frac{\partial \phi_{i,k}}{\partial N_k} \tag{28}$$

Taking the gradient of Eq. (6) in like manner, and using Eq. (26), we get

$$\nabla\Phi\cdot\left(\dot{R}_c - \frac{\nabla\Phi}{2}\right) = \left[\sum_{j=i}^{n} u_j \sum_{k=1}^{3} \phi_{j,k}\right] \cdot \left[\sum_{i=1}^{3} u_i \sum_{k=1}^{3} X_{i,k} N_k - \frac{1}{2}\sum_{i=1}^{n} u_i \sum_{k=1}^{3} \phi_{i,k} N_k\right] \tag{29}$$

Taking the dot product in Eq. (29) yields

$$\nabla\Phi\cdot\left[R_c - \frac{\nabla\Phi}{2}\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} u_i u_j \sum_{k=1}^{3} \phi_{j,k}\left[f_{13_i} X_{i,k} - \frac{1}{2}\phi_{i,k}\right] \tag{30}$$

where

$$f_{n_1 n_{2_i}} = 1 \qquad n_1 \le i \le n_2$$

$$f_{n_1 n_{2_i}} = 0 \qquad i < n_1,\ i > n_2$$

Consider the $\nabla\Phi\cdot(\omega\times\rho)$ term: The angular velocity vectors are referenced to the moving coordinate system. The gradients of the potentials are known in terms of a different coordinate system attached to the patch. To carry out the dot product, we transform the rotation rate vector as follows:

$$\omega = \sum_{i=1}^{3} \omega_i e_i = \sum_{i=1}^{3} \omega_i \sum_{k=1}^{3} X_{i,k} N_k \tag{31}$$

$\nabla\Phi\cdot(\omega\times\rho)$ may then be expressed as

$$\nabla\Phi\cdot(\omega\times\rho) = \sum_{j=1}^{n} u_j \sum_{k=1}^{3} \phi_{j,k} N_k \begin{vmatrix} e_1 & e_2 & e_3 \\ \omega_1 & \omega_2 & \omega_3 \\ X_1 & X_2 & X_3 \end{vmatrix} \tag{32}$$

where

$$e_i = \sum_{k=1}^{3} X_{i,k} N_k \tag{33}$$

Equation (32) is then

$$\nabla\Phi\cdot(\omega\times\rho) = \sum_{j=1}^{n} u_j \sum_{k=1}^{3} \phi_{j,k} N_k \cdot [e_1(\omega_2 X_3 - \omega_3 X_2) + e_2(\omega_3 X_1 - \omega_1 X_3) + e_3(\omega_1 X_2 - \omega_2 X_1)] \tag{34}$$

We note the orderly recursive nature of the indices in Eq. (34) to form the following

$$\nabla\Phi\cdot(\omega\times\rho) = \left[\sum_{j=1}^{n} u_j \sum_{k=1}^{3} \phi_{j,k} N_k\right] \sum_{m=1}^{3} e_m (\omega_{m+1'} X_{m+2'} - \omega_{m+2'} X_{m+1'} \tag{35}$$

where

$$(m+1)' = (m+1) + 3n \tag{36}$$

$$(m+2)' = (m+2) + 3n \tag{37}$$

where n is either a negative, zero, or positive integer which will cause the ( )′ quantity to be either 1, 2, or 3.

Applying Eq. (33) to Eq. (35) gives

$$\nabla\Phi\cdot(\omega\times\rho) = \left[\sum_{j=1}^{n} u_j \sum_{k=1}^{3} \phi_{j,k} N_k\right] \cdot \sum_{m=1}^{3} (\omega_{m+1'} X_{m+2'} - \omega_{m+2'} X_{m+1'}) \sum_{k=1}^{3} X_{m,k} N_k \tag{38}$$

Evaluating the dot product yields

$$\nabla\Phi\cdot(\omega\times\rho) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{3} u_j \phi_{j,k} \cdot [f_{\omega_i}) X_{i,k} (\omega_{i+1'} X_{i+2'} - \omega_{i+2'} X_{i+1'})] \tag{39}$$

where f_{ω_i} is zero, unless i is the index of a unit velocity potential corresponding to an angular rate of a particular body in which case $f_{\omega_i} = 1$. For the motions defined in Eqs. (2) and (3), we get

On the submarine:

$$f\omega_i = 1 \qquad i = 4,6$$

On the missile:

$$f_{\omega_i} = 1 \qquad i = 10,12$$

All other values of f_ω are zero.

Expanding and rearranging Eq. (39) gives

$$\nabla\Phi\cdot(\bar{\omega}\times\bar{\rho})=\sum_{j=1}^{n}\sum_{k=1}^{3}u_j\phi_{j,k}\,[\omega_1\,(X_{3,k}X_2-X_{2,k}X_3)$$

$$+\omega_2\,(X_{1,k}X_3-X_{3,k}X_1)+\omega_3\,(X_{2,k}X_1-X_{1,k}X_2)\,] \qquad (40)$$

which may be expressed

$$\nabla\Phi\cdot(\bar{\omega}\times\bar{\rho})=\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{3}u_j\phi_{j,k}f_{\omega_i}\omega_i$$

$$\cdot\,[X_{i+2',k}X_{i+1'}-X_{i+1',k}X_{i+2'}\,] \qquad (41)$$

Combining Eqs. (30) and (41) into Eq. (18) gives

$$\frac{p}{\rho}=\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{3}\{u_iu_j\phi_{j,k}\,[f_{13_i}X_{i,k}-\tfrac{1}{2}\phi_{i,k}\,]$$

$$+u_j\phi_{j,k}\,f_{\omega_i}\,[X_{i+2',k}X_{i+1'}-X_{i+1',k}X_{i+2'}\,]\,\}-\frac{\mathrm{D}\Phi_p}{\mathrm{D}t} \qquad (42)$$

The $\mathrm{D}\Phi p/\mathrm{D}t$ term is now addressed

$$\Phi=\Phi(X_i,\,Z_j,t) \qquad i=1,3;\;\; j=1,6 \qquad (43)$$

That is, the unit velocity potential distributions, beyond spatial variation relative to the missile frame, also vary at a point on the missile as it emerges from the submarine.

$$\Phi_P=\Phi(Z_j,\,t)j=1,6 \qquad (44)$$

and

$$\frac{\mathrm{D}\Phi_P}{\mathrm{D}t}=\sum_{j=1}^{6}\frac{\partial\Phi_P}{\partial Z_j}\frac{\partial Z_j}{\partial t}+\frac{\partial\Phi}{\partial t} \qquad (45)$$

Using Eq. (6) in Eq. (45)

$$\frac{\mathrm{D}\Phi_P}{\mathrm{D}t}=\sum_{j=1}^{6}\frac{\partial}{\partial Z_j}\sum_{i=1}^{n}u_i\phi_i\frac{\partial Z_j}{\partial t}+\frac{\partial}{\partial t}\sum_{i=1}^{n}u_i\phi_i \qquad (46)$$

which becomes

$$\frac{\mathrm{D}\Phi_P}{\mathrm{D}t}=\sum_{j=1}^{6}\sum_{i=1}^{n}u_i\frac{\partial\phi_i}{\partial Z_j}\frac{\partial Z_j}{\partial t}+\sum_{i=1}^{n}\frac{\partial u_i}{\partial t}\phi_i \qquad (47)$$

Equation (47) gives the nonsteady contribution to pressure due to body acceleration and relative motion. It is the $\partial\phi_i/\partial Z_j$ term that induces rather large numerical computational requirements in emerging body modeling. Potential flow solutions must be determined for a family of relative positions sufficient to determine variational dependence on relative motion. The $\partial Z_j/\partial t$ terms may be transformed into scalar velocities and combined with the u_iu_j coefficients in Eq. (42).

V. Alogrithm Verification

The pressure algorithm, Eq. (42), has been numerically verified. The equation, when developed for an arbitrary body moving with six degrees of freedom in an infinite fluid, takes the form

$$\frac{p}{\rho}=\sum_{i=1}^{6}\Big\{-\dot{u}_i\phi_i+\sum_{j=1}^{6}u_iu_j\sum_{k=1}^{3}[\phi_{j,k}\,(f_{1,3_i}X_{i,k}-\tfrac{1}{2}\phi_{i,k})$$

$$+f_{4,6_i}\,(X_{i+2',k}X_{i+1'}-X_{i+1',k}X_{i+2'}\,)\,]\Big\} \qquad (48)$$

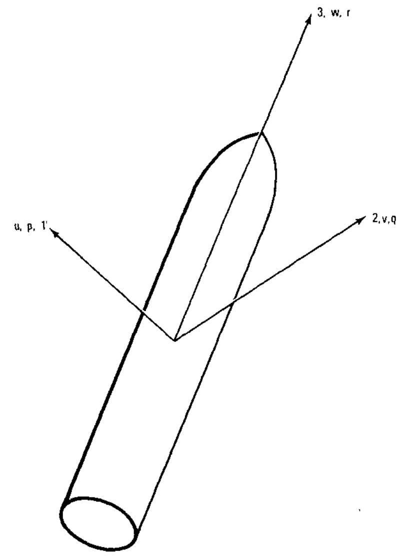

Fig. 4 6 DOF system.

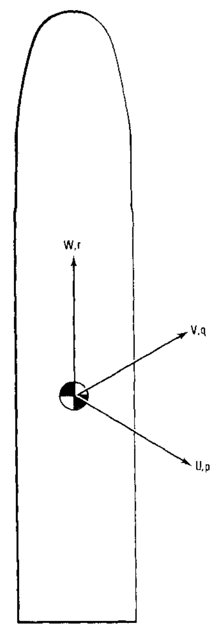

Fig. 5 Test missile profile.

where

$$\begin{Bmatrix}u_1\\u_2\\u_3\\u_4\\u_5\\u_6\end{Bmatrix}=\begin{Bmatrix}U\\V\\W\\p\\q\\r\end{Bmatrix} \qquad (49)$$

as described in Fig. 4.

The lateral force and moment coefficients for a body shown in Fig. 4 are shown in Table 1. Different algorithms were used to generate both the potential flow solutions and the resultant hydrodynamic coefficients. The coefficients agree very well with each other. While only a few coefficients are verified with this particular test, we conclude that the algorithm is correct since it has been used in a general sense, exercising all the elements.

Table 1 Comparison of hydrodynamic coefficients

	C_l	$C5^{\cdot}$	$C13$	$C35$
Normal force	−8.88098[a] (−8.88117)[b]	−3.40609 (−3.40673)	−1.68048 (−1.67895)	2.63502 (2.63816)
Moment	−3.62906 (−3.62971)	−16.49819 (−16.49886)	−5.20096 (−5.27662)	−9.11037 (−9.10038)

[1]Top number derived from infinite fluid, symmetric body code. [b]Bottom number derived from 6 DOF code with new pressure equation.

Acknowledgments

This research was funded in part by the U.S. Department of the Navy, Contract Number N0003074C0127.

References

[1]Burgdorf, O., "SUPERPHI," Westinghouse Electric Corp., Marine Division Tech. Note 76-068, Sept. 1976.

[2]Chow, S.K., Hou, A.Y., and Landweber, L., "Hydrodynamic Forces and Moments Acting on a Body Emerging from an Infinite Plane," *The Physics of Fluids,* Vol. 19, Oct. 1976, pp. 1439-1449.

[3]Burgdorf, O., "Parametric Study of C4 Hydrodynamic Coefficients," Westinghouse Electric Corp., Marine Division Tech. Note 75-058, July 1975.

[4]Lamb, H., "Hydrodynamics," Dover, New York, 1945, 6th ed., p. 20.